# Characterizing the Cognitive Demands of Mathematical Tasks
## A Task-Sorting Activity

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## GOALS

- Raise awareness of how mathematical tasks differ with respect to their levels of cognitive demand
- Highlight the importance of analyzing and discussing tasks in order to determine the level of thinking required to solve them

The point is not that one type of task is better than another; rather, it is important to know the potential of a task so that it can be appropriately mapped on to the goals for students' learning. This activity also raises teachers' awareness of "worthwhile mathematical tasks" as defined by *Professional Standards for Teaching Mathematics* (NCTM 1991).

## CONTENT OF THE TASKS TO BE SORTED

- Number and operation
- Algebra
- Data analysis
- Geometry
- Measurement

GRADE LEVEL: K–12

TIME: 75–90 minutes

## MATERIALS

- Copies of Martha's Carpeting task (transparency 6.1) and the Fencing task (transparency 6.2)
- Tools for working on the two tasks (e.g., calculators, grid paper, square tiles)
- Copies of the sixteen-task card sort at the appropriate grade level (i.e., 3–5, 6–8, 9–12)
- Copies of the recording sheet
- Overhead transparency of the recording sheet
- Overhead transparencies of each task
- Chart paper

## IMPLEMENTATION

Arbaugh and Brown (2004) provide background information on what this task-sorting activity accomplishes and ways that teachers respond when they are asked to complete it.

### Problem Solving (15–20 minutes)

Begin by having teachers work individually on Martha's Carpeting and the Fencing tasks for five minutes and then continue to work on the tasks

with a partner. This gives teachers both a first-hand experience in solving a high-level (Fencing task) and low-level (Martha's Carpeting task) task and a basis for distinguishing between the levels of tasks during the sorting activity.

Orchestrate a whole-group discussion to allow teachers to share various strategies for solving the tasks. Teachers can then discuss the similarities and differences between the two tasks. For example, teachers often note that although both tasks pertain to the same mathematical content (area of a rectangle), they require different types of thinking from the students (with one requiring the application of a formula and the other requiring a deeper level of thinking). The discussion should make salient the fact that all tasks are not created equal—different tasks require different levels and kinds of thinking from students. Tasks that require students to perform a memorized procedure in a routine manner (such a Martha's Carpeting task) lead to one type of opportunity for students' thinking. These are referred to as low-level tasks. Tasks that demand engagement with concepts and that stimulate students to make purposeful connections to meaning or relevant mathematical ideas (such as the Fencing task) lead to a different set of opportunities for students' thinking. These are referred to as high-level tasks.

## Sorting Activity (30 minutes)

Teachers are now ready to begin work on the sorting activity. Working in pairs or triads, have teachers sort the cards into two groups—those that they consider to be high level, and those they consider to be low level—and develop the criteria for each category. As groups finish sorting, have them record their decisions on an overhead. (This way you will be able to see how each group categorizes each task.)

## Discussion of Sorting Activity (30–45 minutes)

Note: You may find it helpful to read Smith and Stein (1998) prior to facilitating the sorting activity so that you are familiar with the characteristics of tasks at both levels.

Orchestrate a whole-group discussion of the sort. Begin by focusing on a task that the majority of the groups characterized as low level. Ask teachers to describe the characteristics of this task while you record their statements on chart paper. Move on to consideration of other tasks that were characterized as low level, adding to and clarifying the criteria as needed. You should then repeat this process for

high-level tasks.

Once the group has come to some agreement regarding a subset of tasks at each level, you are ready to focus on the tasks for which there is some disagreement. For each task, ask teachers whether it is more like the high-level or the low-level tasks, comparing the characteristics on your list. If teachers indicate that the task is low level, ask questions such as "What is the rule or procedure you would use to solve the task?" or "What have you memorized that you are being asked to recall?" If teachers indicated that the task is high level, ask questions such as "What is it you have to think about in order to solve the task?" or "What decisions or judgments do you have to make?"

## Participants' Anticipated Responses

Results from the entire group will most likely show that the teachers disagree about the placement of a number of tasks. Those differences should prompt rich discussions with regard to analysis and characteristics of different levels of tasks. Disagreements often result from making assumptions regarding the cognitive level of a task based on surface features of the task or from equating "high level" with "difficult." For example, some teachers might assume that the feature "requires an explanation" is always associated with tasks with high-level demands. Although many tasks in the sorts are consistent with this view, others can serve as counterexamples to this assertion. For example, in the middle school sort task A (a high-level task) and task N (a low-level task), both require an explanation. The point is to encourage teachers to dig beneath the surface in determining the level of thinking required to complete a task. The following questions should foster lively discussion about these issues.

1. Does a particular feature (e.g., writing an explanation as part of your answer, drawing a picture to explain what you did, using manipulatives to solve the task) indicate that the task has a certain level of cognitive demand?
2. Is there a difference between "level of cognitive demand" and "difficulty"?
3. What effect does context (e.g., setting in which the task is used, students' prior experience, grade level) have on the level of cognitive demand required by a task?

## Answers to the Task-Sorting Activity

"Answers" to the task-sorting activity may be found on the matrices that accompany the tasks.

Tasks categorized as "memorization" and "procedures without connections" are considered to be low level; tasks categorized as "procedures with connections" and "doing mathematics" are considered to be high level. Please see Stein and Smith (1998) for an extended discussion of these four levels of cognitive demand. Note that these answers are for the use of the facilitator in orchestrating the discussion. The goal of this activity is for teachers to participate in a thoughtful analysis of the tasks, not to come to a consensus about the placement of tasks. Also note that the characteristics of the tasks have been identified so as to facilitate the identification of counterexamples (e.g., if a teacher claims that manipulatives are only used in high-level tasks, you can use the matrix to identify a low-level task that also uses manipulatives).

## Further Analysis of Tasks

You may want to have teachers read Smith and Stein (1998) and to differentiate further among tasks using their four categories (i.e., memorization, procedures without connections, procedures with connections, doing mathematics). Teachers could then use these four levels of cognitive demand (Smith and Stein 1998, p. 348) when considering aspects of their classroom practices. The following are suggestions that extend the thinking of the teachers and make connection to their classroom practices.

- Have the teachers collect all the tasks they use in math class for one week. Teachers can then analyze those tasks with regard to the levels of cognitive demand to investigate the level of cognitive demand required of the tasks they use on a daily basis.

- Have teachers analyze their current mathematics text with regard to levels of cognitive demand required by the tasks their students do on a daily basis.

- Have the teachers rewrite tasks that require a low level of cognitive demand to make them into tasks that require a high level of cognitive demand.

## REFERENCES

Arbaugh, Fran, and Catherine A. Brown. "What Makes a Mathematical Task Worthwhile? Designing a Learning Tool for High School Mathematics Teachers." In *Perspectives on the Teaching and Learning of Mathematics*, Sixty-sixth Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Rheta Rebenstein, pp. 27–43. Reston, Va.: NCTM, 2004.

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Stein, Mary Kay, Margaret Schwan Smith, Marjorie A. Henningsen, and Edward A. Silver. *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development*. New York: Teachers College Press, 2000.

Stein, Mary Kay, and Margaret S. Smith. "Mathematical Tasks as a Framework for Reflection: From Research to Practice." *Mathematics Teaching in the Middle School* 3 (January 1998): 268–75.

# Categorizing Mathematical Tasks

(Indicate whether each task is low or high level by placing an X in the appropriate column.)

| TASK | LOW LEVEL | HIGH LEVEL |
|---|---|---|
| A | | |
| B | | |
| C | | |
| D | | |
| E | | |
| F | | |
| G | | |
| H | | |
| I | | |
| J | | |
| K | | |
| L | | |
| M | | |
| N | | |
| O | | |
| P | | |

# Martha's Carpeting Task

Martha was recarpeting her bedroom, which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Source: Stein et al. 2000.

# Fencing Task

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep the rabbits.

1.  If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?

2.  How long would each of the sides of the pen be if they had only 16 feet of fencing?

3.  How would you go about determining the pen with the most room for any amount of fencing?  Organize your work so that someone else who reads it will understand it.

Source: Stein et al. 2000.

# Appendix A

## ELEMENTARY SCHOOL SORTING ACTIVITY

### Matrix of Features Exemplified by Each Task by Category

The purpose of this matrix is to provide a set of counterexamples to assumptions made regarding the categorization of the cognitive level of a task on the basis of surface features of the task.

| Features of Tasks | Memo-rization | | Procedures without Connections | | | | Procedures with Connections | | | | | Doing Mathematics | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | E | L | A | D | G | O | F | I | K | M | P | B | C | H | J | N |
| Uses manipulatives | | | • | | | | | • | • | | • | | | | | • |
| Uses a calculator | | | | | • | | | | | | | • | | | | |
| Uses a diagram | | | • | • | | | • | | • | | | | | • | | |
| Has real-world context | | | | • | | | | • | • | | | | • | • | • | • |
| Is symbolic or abstract | • | • | | | • | • | • | • | | | • | • | • | | | |
| Involves multiple steps, actions, or judgments | | | | | | • | | • | | • | • | • | • | • | | • |
| Requires an explanation | | | | | | • | | • | • | | • | • | | • | • | • |
| Is textbook-like | • | • | | • | • | | • | | • | | | | | | | • |

# TASK A

## Manipulatives or Tools Available: One triangle pattern block

Using the edge of a triangle pattern block as the unit of measure, determine the perimeter of the following pattern-block trains.
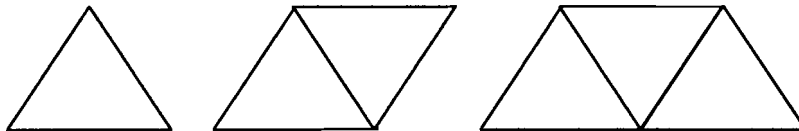


# TASK B

## Manipulatives or Tools Available: Calculator

|  | | Product |
|---|---|---|
| $2 \times 2$ | = | 4 |
| $2 \times 2 \times 2$ | = | 8 |
| $2 \times 2 \times 2 \times 2$ | = | 16 |
| $2 \times 2 \times 2 \times 2 \times 2$ | = | 32 |

If the pattern shown continues, could 375 be one of the products in this pattern? Explain why or why not.

Source: Kenney and Silver 1997, p. 237.

# TASK C

## Manipulatives or Tools Available: None

Think of a real-life situation that describes the following problem.

$$287 \div 14 =$$

Write the problem and then solve it.

# TASK D

**Manipulatives or Tools Available: None**

Use the graph to answer the questions.

## Favorite Colors in Our Class
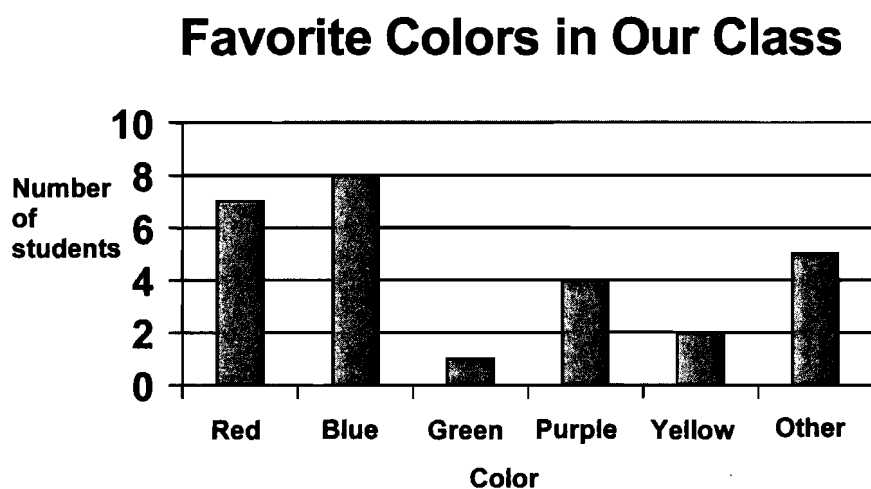


1. How many students picked red as their favorite color?

2. Did more students like blue or purple?

3. Which color did only one student choose as a favorite color?

# TASK E

**Manipulatives or Tools Available: None**

Identify the place value for each of the underlined digits.

1.  35<u>1</u>
2.  <u>7</u>6
3.  <u>4</u>,789
4.  1.<u>2</u>

# TASK F

## Manipulatives or Tools Available: None

About how big is 4/5 of this rectangle? Show your answer by shading in the rectangle.

What other fractions are near 4/5 in size?

Adapted from "Different Shapes, Equal Pieces," in *Investigations in Number, Data, and Space, Grade 4,* by Cornelia Tierney, Mark Ogonowski, Andee Rubin, and Susan Jo Russell, p. 74. Menlo Park, Calif.: Dale Seymour, 1998. Copyright ©1998 by Dale Seymour Publications. Reprinted by permission of Pearson Education, Inc.

# TASK G

## Manipulatives or Tools Available: Calculator

Solve each of the following. Show all your work. Check your answers with a calculator.

1. $8\overline{)96}$      2. $7\overline{)452}$      3. $6\overline{)3288}$

4. $5\overline{)3412}$      5. $10\overline{)4630}$      6. $16\overline{)4952}$

# TASK H

## Manipulatives or Tools Available: None

There are 20 students in Mr. Pang's class. On Tuesday, *most* of the students in the class said they had pockets in the clothes they were wearing.

Which of the graphs below most likely shows the number of pockets each child had?
Explain why you chose that graph and why you did not choose the other graphs.



**A**

$= 1$ Student

**B**

$= 1$ Student

**C**

$= 1$ Student
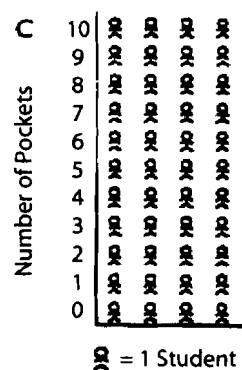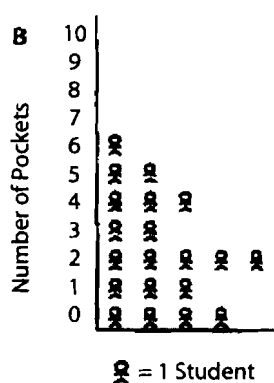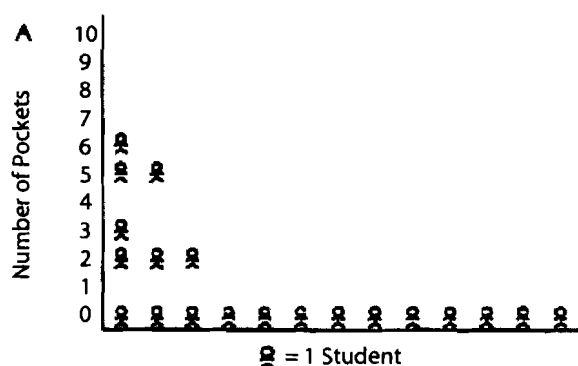
# Task I

## Manipulatives or Tools Available: Interlocking cubes, grid paper

Solve the two sets of problems shown below. You can use interlocking cubes or the grid paper to make arrays to help you solve these problems. Try to solve the last problem in each set by thinking about the other problems in the set. You can add problems to the set that help you solve the final problem better. After you solve the problems, write about how you solved the last problem in each set.

| |
|---|
| 10 × 6 |
| 3 × 6 |
| 6 × 6 |
| 13 × 6 |

| |
|---|
| 4 × 6 |
| 4 × 10 |
| 4 × 12 |
| 4 × 30 |
| 4 × 36 |

Adapted from "Arrays and Shares," in *Investigations in Number, Data, and Space, Grade 4,* by Cornelia Tierney, Mark Ogonowski, Andee Rubin, and Susan Jo Russell, p. 110. Menlo Park, Calif.: Dale Seymour, 1998. Copyright ©1998 by Dale Seymour Publications. Reprinted by permission of Pearson Education, Inc.

# Task J

## Manipulatives or Tools Available: None

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all your work.

Jose ate 1/2 of a pizza.

Ella ate 1/2 of another pizza.

Jose said that he ate more pizza than Ella, but Ella said they both ate the same amount.
Use words and pictures to show that Jose could be right.

Source: Kenney and Silver 1997, p. 119.

# TASK K

## Manipulatives or Tools Available: Cubes

Use cubes to model the situations below. Write a number sentence for each problem. Then use words or pictures to explain how you solved each problem.

1. If each student is to receive 3 pencils, how many students will 15 pencils supply?

2. Jamie and Alex baked 48 cookies for the bake sale and packaged them in groups of 6 cookies per bag. How many bags did they pack?

3. If 32 cookies are divided equally among 8 students, how many cookies will each student receive?

Adapted from *Visual Mathematics Course Guide, Volume 1.* Salem, Ore.: Math Learning Center, 1995, Lesson 2, "Basic Operations."

# TASK L

## Manipulatives or Tools Available: None

Complete the following multiplication facts in one minute or less.

| | | |
|---|---|---|
| $2 \times 3 =$ _____ | $5 \times 4 =$ _____ | $10 \times 6 =$ _____ |
| $4 \times 7 =$ _____ | $8 \times 10 =$ _____ | $8 \times 4 =$ _____ |
| $9 \times 5 =$ _____ | $3 \times 4 =$ _____ | $5 \times 5 =$ _____ |
| $6 \times 8 =$ _____ | $7 \times 9 =$ _____ | $2 \times 6 =$ _____ |
| $3 \times 9 =$ _____ | $8 \times 7 =$ _____ | $9 \times 2 =$ _____ |

# TASK M

## Manipulatives or Tools Available: None

Anita has four 20-point projects for science class. Her scores are shown below. What is her average score? Find the average for Anita's scores by leveling off the stacks.
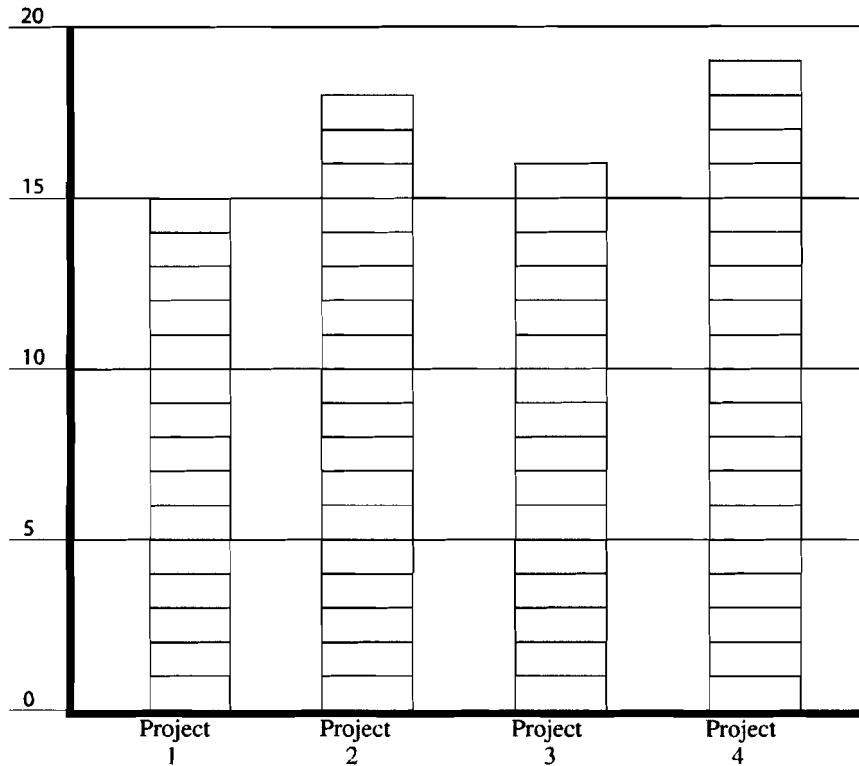


Adapted from *QUASAR Cognitive Assessment Instrument*.

# TASK N

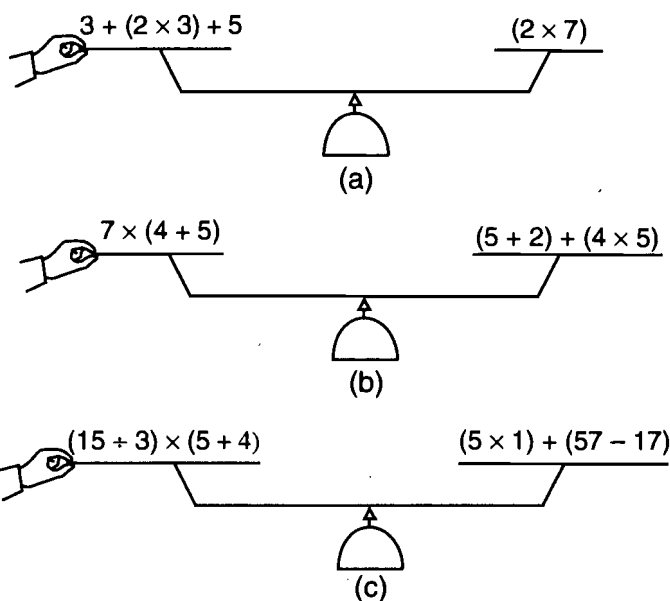## Manipulatives or Tools Available: Grid paper, interlocking cubes

1. The kindergarten class is coming to watch a play in our classroom. There are 20 students. In what different ways could we arrange the chairs for them so that all the rows are equal?

2. The two third-grade classes are going to watch our play in the cafeteria. There are 49 students altogether. In what different ways would we arrange the chairs for them so that all the rows are equal?

3. What do you notice about your solutions for problem 1 and problem 2?

From *Investigations in Number, Data, and Space, Grade 4*, by Cornelia Tierney, Mark Ogonowski, Andee Rubin, and Susan Jo Russell, p. 75. Menlo Park, Calif.: Dale Seymour, 1998. Copyright ©1998 by Dale Seymour Publications. Reprinted by permission of Pearson Education, Inc.

# TASK O

## Manipulatives or Tools Available: None

Tell if the scale will balance (equation) or tilt (inequality). If it does not balance, please write which side will tilt down.

$3 + (2 \times 3) + 5$      $(2 \times 7)$

(a)

$7 \times (4 + 5)$      $(5 + 2) + (4 \times 5)$

(b)

$(15 \div 3) \times (5 + 4)$      $(5 \times 1) + (57 - 17)$

(c)

Write a reason for your answer.
Source: Cuevas, Yeatts, and House 2001, p. 81.

# TASK P

## Manipulatives or Tools Available: Counters, cubes, grid paper, base-ten blocks

Solve this problem in two different ways:

$$32 - 17 = \underline{\hspace{1cm}}$$

After each way, write about how you did it. Be sure to include

- what materials, if any, you used to solve this problem;
- how you solved it;
- an explanation of your thinking as you solved it.

First Way:

Second Way:

Adapted from "Packages and Groups," in *Investigations in Number, Data, and Space, Grade 4*, by Cornelia Tierney, Mark Ogonowski, Andee Rubin, and Susan Jo Russell, p. 84. Menlo Park, Calif.: Dale Seymour, 1998. Copyright ©1998 by Dale Seymour Publications. Reprinted by permission of Pearson Education, Inc.

## MIDDLE SCHOOL SORTING ACTIVITY

### Matrix of Features Exemplified by Each Task by Category

The purpose of this matrix is to provide a set of counterexamples to assumptions made regarding the categorization of the cognitive level of a task on the basis of surface features of the task.

| Features of Tasks | Memo-rization | | Procedures without Connections | | | | Procedures with Connections | | | | | Doing Mathematics | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | F | O | C | E | I | N | D | H | J | L | P | A | B | G | K | M |
| Uses manipulatives | | | • | | | | | | | • | • | | • | • | | |
| Uses a calculator | | | | • | | | | | | | | • | | | • | |
| Uses a diagram | | | • | | | | | • | • | | • | | • | | | • |
| Has real-world context | | | | | | • | • | | | | | • | | | • | • |
| Is symbolic or abstract | | | | • | • | | • | • | • | • | | | • | • | | |
| Involves multiple steps, actions, or judgments | | | | • | • | • | • | • | • | • | • | • | • | • | • | • |
| Requires an explanation | | | | • | | | • | | | | | • | • | • | • | • |
| Is textbook-like | • | • | | • | • | • | • | | • | | | • | | | | |

# Task A

## Manipulatives or Tools Available: Calculator

Treena won a 7-day scholarship worth $1,000 to the Pro Shot Basketball Camp. Round-trip travel expenses to the camp are $335 by air or $125 by train. At the camp she must choose between a week of individual instruction at $60 a day or a week of group instruction at $40 A day. Treena's food and other expenses are fixed at $45 a day. If she does not plan to spend any money other than the scholarship, what are *all* choices of travel and instruction plans she could afford to make? Explain which option you think Treena should select and why.
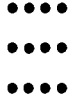
Source: Kenney and Silver 1997, p. 108.

# Task B

## Manipulatives or Tools Available: Counters

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough for another person to read it and understand your thinking. It is important that you show *all* your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.

| (1st step) | (2nd step) | (3rd step) |
|:---:|:---:|:---:|
| | | ● ● ● ● |
| | ● ● ● | ● ● ● ● |
| ● ● | ● ● ● | ● ● ● ● |
| 2 dots | 6 dots | 12 dots |

Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.
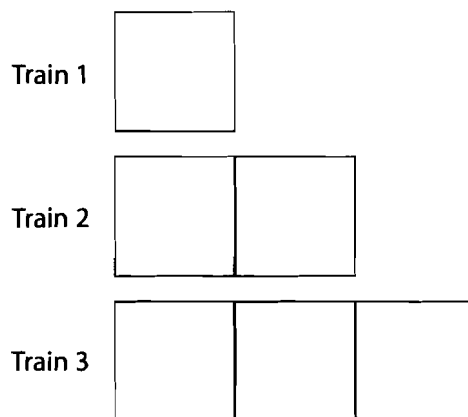
Explain how she could do this *and* give the answer that Marcy should get for the number of dots.

Source: Kenney and Silver 1997, p. 240.

# TASK C

## Manipulatives or Tools Available: Square pattern tiles

Using the side of a square pattern tile as a measure, find the perimeter (i.e., distance around) of each train in the pattern block figure shown below.

Train 1

Train 2

Train 3

# TASK D

## Manipulatives or Tools Available: None

Part A: The place kicker on the North High School football team has made 13 out of 20 field goals so far this season. The place kicker on the South High football team has made 15 out of 25 field goals so far this season. Which player has made the greatest percentage of field goals?

Part B: If the "better" player does not play for the rest of the season, how many field goals would the other player have to make in the next 10 attempts to have the greatest percentage of field goals?

# TASK E

## Manipulatives or Tools Available: Calculator

Divide using paper and pencil. Check your answer with a calculator and round the decimal to the nearest thousandth.

$$\frac{525}{1.3}$$

$$\frac{52.75}{7.25}$$

$$\frac{30.459}{.12}$$

# TASK F

## Manipulatives or Tools Available: None

Match the property name with the appropriate equation.

| | | | | |
|---|---|---|---|---|
| 1. | Commutative property of addition | a. | $r(s + t) = rs + rt$ |
| 2. | Commutative property of multiplication | b. | $x \cdot 1/x = 1$ |
| 3. | Associative property of addition | c. | $-y + x = x + (-y)$ |
| 4. | Associative property of multiplication | d. | $a/b + -a/b = 0$ |
| 5. | Identity property of addition | e. | $y \cdot (zx) = (yz) \cdot x$ |
| 6. | Identity property of multiplication | f. | $1 \cdot (xy) = xy$ |
| 7. | Inverse property of addition | g. | $d \cdot 0 = 0$ and $0 \cdot d = 0$ |
| 8. | Inverse property of multiplication | h. | $x + (b + c) = (x + b) + c$ |
| 9. | Distributive property | i. | $y + 0 = y$ |
| 10. | Property of zero for multiplication | j. | $p \cdot q = q \cdot p$ |

# TASK G

## Manipulatives or Tools Available: Base-ten blocks, grid paper

.08      .8      .080      .008000

Make three observations about the relative size of the four numbers above. Be sure to explain your observations as clearly as possible. Feel free to illustrate your observations if you think it would help others understand them.

Adapted from QUASAR Project—*QUASAR Cognitive Assessment Instrument*—Released Task.

# TASK H

## Manipulatives or Tools Available: Grid Paper

The pairs of numbers in (*a*) through (*d*) below represent the heights of stacks of cubes to be leveled off. On grid paper, sketch the front views of columns of cubes with these heights before and after they are leveled off. Write a statement under the sketches that explains how your method of leveling off is related to finding the average of the two numbers.

*a*) 14 and 8      *b*) 16 and 7      *c*) 7 and 12      *d*) 13 and 15

*By taking 2 blocks off the first stack and giving them to the second stack, I've made the two stacks the same. So the total number of cubes is now distributed into 2 columns of equal height. And that is what average means.*

Taken from *Visual Mathematics, Course 1,* The Math Learning Center, 1995, Lesson 10, Follow-up Student Activity 10.1, #1, p. 121.

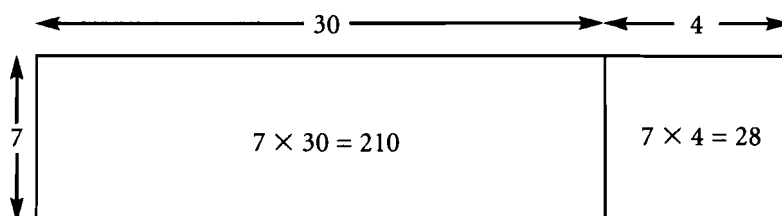# TASK I

## Manipulatives or Tools Available: None

Write and solve a proportion for each of the following.

17 is what percent of 68?
What is 15% of 60?
8 is 10% of what number?
24 is 25% of what number?
28 is what percent of 140?
What is 60% of 45?
36 is what percent of 90?
What is 80% of 120?
21 is 30% of what number?

# Task J

## Manipulatives or Tools Available: None

One method of mentally computing 7 × 34 is illustrated in the diagram below.

$$\xleftarrow{\hspace{3cm}} 30 \xrightarrow{\hspace{3cm}} \xleftarrow{} 4 \xrightarrow{}$$

| | |
|---|---|
| 7 × 30 = 210 | 7 × 4 = 28 |

(with a vertical 7 labeling the height on the left)

Mentally compute these products. Then sketch a diagram that describes your methods for each.

a)      27 × 3

b)      325 × 4

Adapted from *Visual Mathematics, Course 1*. The Math Learning Center, 1995, Lesson 37, Focus Teacher Activity, pp. 457–63.

# Task K

## Manipulatives or Tools Available: Calculator with scientific functions

Penny's mother told her that several of her great-great-great-grandparents fought in the Civil War. Penny thought this was interesting, and she wondered how many great-great-great-grandparents that she actually had. When she found that number, she wondered how many generations back she'd have to go until she could count more than 100 ancestral grandparents or 1000, or 10,000, or even 100,000. When she found out, she was amazed and pretty glad she had a calculator. How do you think Penny might have figured out all this information? Explain and justify your method as clearly and completely as possible.

Adapted from Blackline Master 3B from *AW Mathematics: Mathematics with Calculators: Resources for Teachers, Grade 6*, Section 3, Activity 2. Addison-Wesley Publishing Co., 1988. Copyright ©1988 Addison-Wesley Publishing Co., Inc. Reprinted by permission of Pearson Education, Inc.

# Task L

## Manipulatives or Tools Available: Base-ten blocks

Using base-ten blocks, show that 0.292 is less than 0.3.

# TASK M

## Manipulatives or Tools Available: None

Use the following information and the graph to write a story about Tony's walk:

At noon, Tony started walking to his grandmother's house. He arrived at her house at 3:00. The graph below shows Tony's speed in miles per hour throughout his walk.
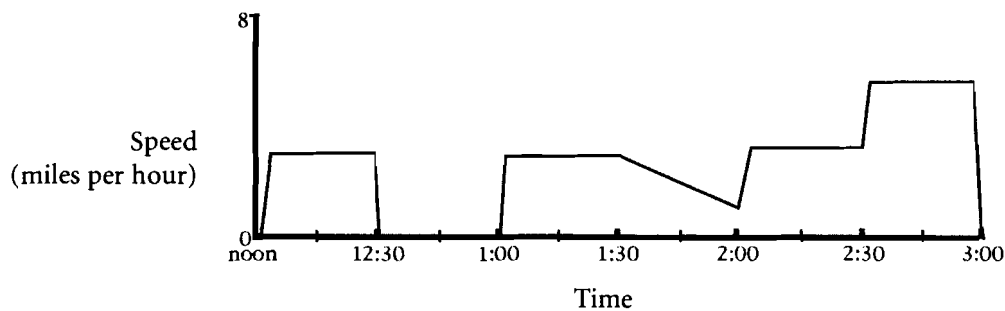


Write a story about Tony's walk. In your story, describe what Tony might have been doing at the different times.

Taken from the QUASAR Project—*QUASAR Cognitive Assessment Instrument*—Released task.

# TASK N

## Manipulatives or Tools Available: None

The cost of a sweater at J. C. Penney's was $45.00. At the "Day and Night Sale" it was marked 30% off the original price. What was the price of the sweater during the sale? Explain the process you used to find the sale price.

# TASK O

## Manipulatives or Tools Available: None

Give the fraction and percent for each decimal.

0.20 = _____ = _____

0.25 = _____ = _____

0.3̄3̄ = _____ = _____

0.50 = _____ = _____

0.6̄6̄ = _____ = _____

0.75 = _____ = _____

# Task P

## Manipulatives or Tools Available: Pattern blocks

Find 1/2 of 1/3. Use pattern blocks. Draw your answer.

1/2 of 1/3 or 1/2 × 1/3 = _____

Find 1/3 of 1/4. Use pattern blocks. Draw your answer.

1/3 of 1/4 or 1/3 × 1/4 = _____

Find 1/4 of 1/3. Use pattern blocks. Draw your answer.

1/4 of 1/3 or 1/4 × 1/3 = _____

# Appendix C

## HIGH SCHOOL SORTING ACTIVITY

### Matrix of Features Exemplified by Each Task by Category

The purpose of this matrix is to provide a set of counterexamples to assumptions made regarding the categorization of the cognitive level of a task on the basis of surface features of the task.

| Features of Tasks | Memorization | | Procedures without connections | | | | | | Procedures with connections | | | | | Doing Mathematics | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | C | K | B | D | F | G | H | N | E | I | L | M | O | A | J | P |
| Could use manipulatives | | | • | | | | | | | | | • | • | | | |
| Could use calculator | | | | | • | • | • | • | • | • | • | • | • | • | • | • |
| Uses/requires a diagram | | | | | • | | • | • | • | | | • | • | | | • |
| Has real-world context | | | | • | • | | | | • | | | • | • | • | | |
| Is symbolic or abstract | • | • | • | | • | • | • | • | • | • | • | | • | | | |
| Involves multiple steps, actions, or judgments | | | | | • | • | • | | • | • | • | • | • | • | • | • |
| Requires an explanation | | | | | • | | | | • | • | | | | | • | |
| Is textbook-like | • | • | • | • | • | • | • | • | • | | | • | • | | | |

# TASK A

Find the smallest positive integer that has *exactly* 13 factors.

# TASK B

Factor the following polynomials.

1. $x(x + 1) - 3(x + 1)$
2. $x^2 + 5x + 6$
3. $4x^2 - 25$
4. $27x^3 + 8$

# TASK C

State the triangular and unit circle definitions for sin $\theta$, cos $\theta$, and tan $\theta$.

# TASK D

Your math class has a ratio of boys to girls that is 2:3. If there are 8 boys in your math class, how many girls are there?

# TASK E

Biologists have determined that the polynomial function

$$p(t) = -0.0001t^3 + 0.002t^2 + 1.5t + 100$$

approximates the population $t$ days later of a certain group of wild turkeys left to reproduce on their own with no predators.

*a)* Draw a complete graph of the algebraic model $y = p(t)$ of this problem situation.

*b)* Find the maximum turkey population and when it occurs. Explain how you know this is the maximum population.

*c)* When will the turkey population be extinct? Explain how you know this date.

Source: Demana, Waits, and Clemens, *Precalculus,* (adapted from p. 184) © 1994. Reprinted by permission of Pearson Education, Inc. Publishing as Pearson Addison Wesley.

# TASK F

Find the value of $x$ in the figure below. Write a paragraph that explains how you found $x$.
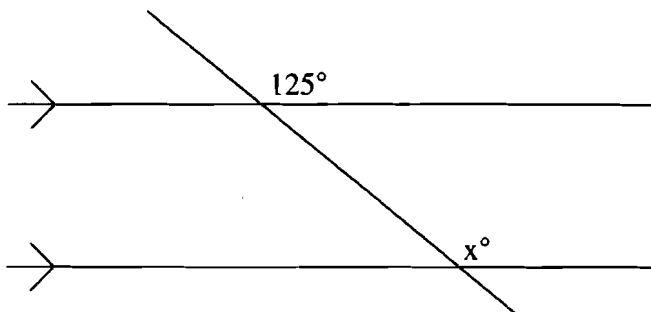
# TASK G

A will states that John is to get 3 times as much money as Mary. The total amount they will receive is $11,000.

1. Write a system of equations describing this situation.
2. Solve to find the amounts of money John and Mary get.

# TASK H

Insert parentheses to make each statement true.

1. $2 \times 14 - 9 - 17 - 14 = 7$
2. $16 + 5 \times 4 \div 2 = 42$
3. $64 \div 8 + 24 - 1 = 1$
4. $36 \div 3 - 9 \div 3 = 1$

# TASK I

Solve this equation by factoring.

$$x^2 - 7x + 12 = 0$$

Explain how the factors of the equation relate to the root of the equation and how you could use that information to draw a sketch of the parabola. Then draw the sketch.

# TASK J

Postal rates have been figured by the ounce since July 1, 1885. From that date until January 1, 1995, the rates have been as follows.

| | | |
|---|---|---|
| Nov. 3, 1917 | 3 | cents |
| July 1, 1919 | 2 | cents |
| July 6, 1932 | 3 | cents |
| Aug. 1, 1953 | 4 | cents |
| Jan. 7, 1963 | 6 | cents |
| May 16, 1971 | 8 | cents |
| March 2, 1974 | 10 | cents |
| Dec. 31, 1975 | 13 | cents |
| May 29, 1978 | 15 | cents |
| March 22, 1981 | 18 | cents |
| Nov. 1, 1981 | 20 | cents |
| Feb. 17, 1985 | 22 | cents |
| April 3, 1988 | 25 | cents |
| Feb. 3, 1991 | 29 | cents |

On the basis of the data above, predict the cost of mailing a one-ounce, first-class letter in 2010.
Explain your reasoning.

# TASK K

Match the following rule to its correct name:

1. $a + b = b + a$
2. $(a + b) + c = a + (b + c)$
3. $a(b + c) = ab + ac$
4. $a + 0 = a$
5. $a(1) = a$
6. If $a = b$, and $b = c$, then $a = c$

a. Identity property for multiplication
b. Commutive property of addition
c. Transitive property
d. Associative property of addition
e. Identity property for addition
f. Distributive property

# TASK L

Use the table of values below to draw a graph of the function represented. Then use the graph to write the equation of the function. Then use the equation of the function to find $f(5)$, $f(-11)$, and $f(1/2)$. Also use the function to find $x$ if $f(x) = 218$.

| $x$ | $f(x)$ |
|-----|--------|
| 1   | −1     |
| 3   | 3      |
| 0   | −3     |
| −2  | −7     |

# TASK M

A 25-foot ladder is placed against a building. The bottom of the ladder is 7 feet from the building. If the top of the ladder slips down 4 feet, how many feet will the bottom slide out? No, it is not 4 feet. This is a two-step problem, so draw two right triangles.

Source: Serra 1993, p. 396. *Discovering Geometry: An Inductive Approach*, Key Curriculum Press, 1150 65th Street, Emeryville, CA 94608, 1-800-995-MATH, www.keypress.com.

# TASK N

Find $x$.
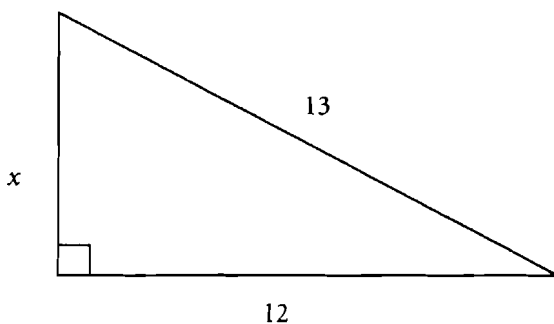
# TASK O

In 1919, Babe Ruth hit the longest home run ever recorded in major-league baseball. In an exhibition game between the Boston Red Sox and the New York Giants, he sent the ball into a parabolic arc. The trajectory of the ball is given by the equation $y = x - 0.0017x^2$, where $x$ represents the horizontal distance (in feet) and $y$ the vertical distance (in feet) of the ball from home plate. Use your graphing calculator (the graph and calculate buttons) to answer the following questions.

1. What was the greatest height reached by the ball?
2. How far from home plate did the ball land?

# TASK P

Write the next two rows in this pattern.

```
                1
             3     5
          7     9     11
       13    15    17    19
    21    23    25    27    29
```